

MICROCOPY RESOLUTION TEST CHART



	REPORT DOCU	MENTATION	PAGE			
'a REPORT SECURITY CLASSIFICATION	'b RESTRICTIVE MARKINGS					
Unclassified						
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION AVAILABILITY OF REPORT				
26 DECLASSIFICATION DOWNGRADING SCHEDULE		Unlimited				
4 PERFORMING ORGANIZATION REPORT NUMBER(S)		S MONITORING ORGANIZATION REPORT NUMBER(S)				
Cornell University TR 85-70	8					
NAME OF PERFORMING ORGANIZATION	60 OFFICE SYMBOL	7a NAME OF MONITORING ORGANIZATION				
Cornell University	(If applicable)	Office of Naval Research				
ADDRESS City State, and ZIP Code)		7b ADDRESS (City, State, and ZIP Code)				
Dept. of Computer Science		800 North Quincy Street				
Cornell University <u>Ithaca, NY</u> 14853	Arlington, VA 22217-5000					
NAME OF FUNDING SPONSORING	86 OFFICE SYMBOL	9 PROCUREMENT INSTRUMENT DENTIFICATION NUMBER NO0014-86-K-0092				
DRGANIZATION Office of Nevel Beggarah	(If applicable)					
Office of Naval Research ODRESS (City, State, and ZIP Code)		10 SOURCE OF FUNDING NUMBERS				
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Arlington, VA 22217-5000		ELEMENT NO	NO	VO		ACCESSION NO
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12 PERSONAL AUTHOR(S)						
Bowen Alpern	, Alan J. Demers	, Fred B. Sch	nneider			
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	concurrent	program's, program properties				
19 ABSTRACT (Continue on reverse if necessal	y and identify by block	number)				
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22a NAME OF RESPONSIBLE INDIVIDUAL Fred B. Schneider	226 TELEPHONE		ide) 22c	OFFICE SY	MBOL	

Safety without Stuttering*

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> TR 85-708 October 1985

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This work is supported, in part, by NSF Grant DCR-8320274 and a grant from the Office of Naval Research.

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Safety without Stuttering

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ABSTRACT

A new formalization of safety properties is given. The formalization agrees with the informal definition—that a safety property stipulates that some "bad thing" doesn't happen during execution—for properties that are not invariant under stuttering, as well as for properties that are.

This work is supported, in part, by NSF Grant DCR-8320274 and a grant from the Office of Naval Research.

1. Introduction

Informally, a safety property stipulates that some "bad thing" doesn't happen during execution [Lamport 77]. Examples of safety properties include mutual exclusion, deadlock freedom, and partial correctness. In mutual exclusion, the proscribed "bad thing" is two processes executing in critical sections at the same time. In deadlock freedom, it is deadlock. In partial correctness, it is terminating in a state not satisfying the postcondition when execution is started in a state that satisfies the precondition.

A formal definition of safety is given in [Lamport 85]. While that definition correctly captures the intuition for an important class of properties—those invariant under stuttering—it is inadequate for safety properties that are not invariant under stuttering. This note gives a formal definition of safety that is independent of stuttering.

Section 2 of the paper reviews some notation for describing properties. Section 3 gives our new formalization of safety and relates it to the one in [Lamport 85]. Finally, section 4 puts our work into perspective.

2. Properties

An execution of a concurrent program can be described by an infinite sequence of states $\sigma = s_0 s_1 \dots$

which we call a history. Each state after s_0 results from executing a single atomic action in the preceding state. For a terminating program execution, an infinite sequence is obtained by repeating the final state. This corresponds to the view that a terminating execution is the same as a non-terminating execution in which after some finite time (once the program has terminated) the state remains fixed.

A property is a set of histories. We write $\sigma \vdash P$ to denote that history σ is a member of property P. A property is usually defined by a characteristic predicate on histories rather than by enumerating the histories themselves. Temporal logic provides a suitable formalism for this purpose [Lamport 83].

The following notation is used in the remainder of the paper. S is the set of states, S° the set of finite sequences of states, and S^{ω} the set of histories. For a history $\sigma = s_0 s_1 ...$, define

$$\sigma[i] = s_i$$

$$\sigma[..i] = s_0 s_1 ... s_i$$

$$\sigma[i..] = s_i s_{i+1} ...$$

We use superscripts to denote repetition. Thus, for α in S^* , α^n is the finite sequence obtained by repeating α n times and α^{ω} is the history obtained by repeating α indefinitely. We use juxtaposition to denote catenation of state sequences.

3. Formalizing Safety

If a "bad thing" happens in a history, then it must do so in some finite prefix of that history. Based on this, Lamport [Lamport 85] formalized a safety property as any property P satisfying

$$SP_L(P)$$
: $(\forall \sigma: \sigma \in S^\omega: \sigma \models P \Leftrightarrow (\forall i: 0 \le i: \sigma[..i]\sigma[i]^\omega \models P))$

Thus, a safety property P is satisfied by a history σ if and only if every prefix of σ —extended to an infinite sequence by repeating its last state—also satisfies P. Extension of a finite sequence $(\sigma[...l])$ to an infinite one is necessary because only a history can satisfy a property; repetition of the last step is one of a number of ways to perform this extension.

For some properties, extending a finite sequence by repeating the final state causes problems. Consider property *CP* stipulating that a variable *clock* is increased for every instruction executed. Using the temporal logic notation "O" for the "next-time" operator, this is given by

CP:
$$(clock = N) \Rightarrow \bigcirc (clock > N)$$
.

Intuitively, CP is a safety property: the "bad thing" is clock not increasing in two successive states. However, CP does not satisfy the formal definition of safety given above. $SP_L(CP) = false$ because for no history σ —even if $\sigma \models CP$ —will the value of clock change after the i^{th} state in $\sigma[...l] \sigma[i]^{\omega}$.

This difficulty arises only for properties that are not invariant under stuttering. A property is invariant under stuttering if and only if whenever a history satisfies the property, the history with every state repeated zero or more times also satisfies the property, and vice versa. More formally, any property P satisfying

STR(P):
$$(\forall f: f \in \mathbb{N} \rightarrow \mathbb{N}: \sigma \models P \Leftrightarrow \sigma[0]^{f(0)+1} \dots \sigma[i]^{f(i)+1} \dots \models P)$$

is invariant under stuttering. Properties that are invariant under stuttering are well suited for hierarchical specification and verification [Lamport 83]. By permitting states to be repeated, meaningful statements can be made about the system at various levels of abstraction. For example, execution of a higher-level operation that is implemented by a sequence of lower-level operations can be viewed as a sequence of repeated, identical, higher-level states where there is one state for every lower-level instruction executed but the last, which produces a new higher-level state.

We now give a formalization of safety that agrees with SP_L for properties invariant under stuttering and that agrees with the informal definition of safety for properties (like CP)

that are not. If a safety property P does not hold for a history σ , then some "bad thing" must have happened during σ . This "bad thing" must be irremediable, because a safety property requires that the "bad thing" never happen. Thus, if $\neg(\sigma \models P)$, there is some prefix of σ (that includes the "bad thing") for which no extension to a history will satisfy P. Taking the contrapositive of this, P is a safety property if it satisfies

```
SP_{ADS}(P): (\forall \sigma: \sigma \in S^{\omega}: \sigma \models P \Leftrightarrow (\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: \sigma[..i]\beta \models P))).
```

 SP_{ADS} differs from SP_L in the way prefixes are extended to form histories. SP_{ADS} permits extension using any history β , while SP_L requires extension by replicating the last state of the prefix. Note that $SP_{ADS}(CP) = true$, so CP is a safety property according to this formalization.

The relationship between SP_L and SP_{ADS} is given in the following two theorems. The first theorem states that safety properties under SP_L are also safety properties under SP_{ADS} .

Theorem: For any property P, $SP_L(P) \Rightarrow SP_{ADS}(P)$.

Proof: Assuming $SP_L(P)$, we must show $\sigma \models P \Leftrightarrow (\forall i: 0 \le i: (\exists \beta: \beta \in S^\omega: \sigma[..i]\beta \models P))$.

```
(\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: \sigma[..i]\beta \models P))
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- \Rightarrow $(\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: \sigma[..i] \sigma[i]^{\omega} \models P))$ $SP_L(P)$, since $\sigma[..i] \beta \models P$
- \Leftrightarrow $(\forall i: 0 \le i: \sigma[..i] \sigma[i]^{\omega} = P)$ by Predicate Logic
- $\Leftrightarrow \sigma \models P \quad \text{by } SP_L(P).$

The second theorem states that every safety property according to SP_{ADS} that is invariant under stuttering is also a safety property according to SP_L .

Theorem: For any property P, $(SP_{ADS}(P) \land STR(P)) \Rightarrow SP_L(P)$.

Proof: Assuming $SP_{ADS}(P)$ and STR(P), we must show:

- (1) $\sigma \models P \Rightarrow (\forall i: 0 \le i: \sigma[..i] \sigma[i]^{\omega} \models P)$
- (2) $(\forall i: 0 \le i: \sigma[..i]\sigma[i]^{\omega} \models P) \Rightarrow \sigma \models P$

First, we prove (1):

```
\sigma \models P
(*) \Leftrightarrow (\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: \sigma[..i] \beta \models P)) by SP_{ADS}(P)
\Leftrightarrow (\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: (\forall n: 0 \le n: \sigma[..i] \sigma[i]^n \beta \models P))) by STR(P)
\Leftrightarrow (\forall i: 0 \le i: (\forall n: 0 \le n: (\exists \beta: \beta \in S^{\omega}: \sigma[..i] \sigma[i]^n \beta \models P))) by Predicate Logic
\Leftrightarrow (\forall i: 0 \le i: (\forall n: 0 \le n: (\exists \beta: \beta \in S^{\omega}: (\sigma[..i] \sigma[i]^{\omega})[..i + n]
\Leftrightarrow (\forall i: 0 \le i: (\forall f: i \le f: (\exists \beta: \beta \in S^{\omega}: (\sigma[..i] \sigma[i]^{\omega})[..f] \beta \models P))) by Predicate Logic
\Leftrightarrow (\forall i: 0 \le i: (\forall f: 0 \le f: (\exists \beta: \beta \in S^{\omega}: (\sigma[..i] \sigma[i]^{\omega})[..f] \beta \models P)))
\text{since } j < i \Leftrightarrow (\sigma[..i] \sigma[i]^{\omega})[..\sigma[i]^{\omega}] and according to (*), \sigma[..f] \beta \models P)
\Leftrightarrow (\forall i: 0 \le i: \sigma[..i] \sigma[i]^{\omega} \models P) since SP_{ADS}(P).
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Next, we prove (2):
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(\forall i: 0 \le i: \sigma[..i] \sigma[i]^{\omega} \models P)
\Rightarrow (\forall i: 0 \le i: (\exists \beta: \beta \in S^{\omega}: \sigma[..i] \beta \models P)) \quad \text{use } \beta = \sigma[i]^{\omega}
\Rightarrow \sigma \models P \quad \text{by } SP_{ADS}(P).
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4. Discussion

It has been argued that properties invariant under stuttering are the only ones of real interest in program verification [Lamport 83]. We agree. This, however, is a religious issue. A formalization of safety should serve many faiths. This note presents a definition of safety that can be applied to any property.

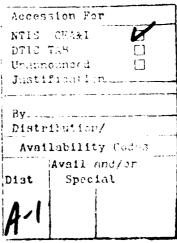
Acknowledgments

Suggestions by D. Gries led to improvements in the clarity of the proofs.

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